Theoretical approaches, progress and directions

Nuclear theorists attempt to correlate known data by practical models and to predict the behavior of nuclear systems in new regimes. Theory plays an important role in the analysis of experiments, and in suggesting what new data would be valuable in future work.

Microscopic nuclear models continue to improve due to advances in theoretical techniques and computational resources and to improvements in the data that are used to constrain the input parameters. Exact theoretical solutions are still not possible for most nuclei, but one can obtain essentially exact solutions for systems containing up to eight nucleons, and very good results for infinite nuclear matter. The precise data on nucleon-nucleon scattering are used as input, and three-body interactions are also important. Most of the reaction rates for big-bang nucleosynthesis and solar neutrino production can be addressed by these ab initio calculations. Faddeev and hyperspherical harmonic methods give exact 3- and 4-body scattering solutions today, and Green's function Monte Carlo methods are promising for larger nuclei, up to at least A = 12. Since most low-energy reactions occur at large radii, further work is needed to incorporate appropriate asymptotic forms for the bound and continuum wave functions. These exact methods are also important for understanding the validity and limitations of the more approximate many-body techniques that must be used for heavier nuclei.

For the entire range of larger nuclei one relies upon mean field and shell-model configuration mixing methods. Both of these methods have recently been refined and improved. Of the mean field models, the energy-density functional methods have been most successful. For example, in the Skyrme-Hartree-Fock method the nuclear properties are related to a few (6-12) parameters of a density-dependent interaction that are determined by fitting nuclear data. This method, together with quasiparticle RPA calculations of the Gamow-Teller strength functions, provides an initial assessment of the global mass, beta-decay, and electron capture properties that are needed to describe supernova-core collapse, the r-process, and rp-process. The beta-decay half-lives are determined by the small part of the Gamow-Teller resonance that lies at low excitation energy. This aspect of the calculations needs to be improved. Hartree-Fock might be most appropriate for spherical and deformed regions, but for intermediate regions, the generator coordinate method or its equivalent should be used.

Large-basis shell-model calculations provide a more complete picture, in principle, but have been limited by the large dimensions of the valence spaces encountered in typical astrophysics applications. However, the newly developed quantum Monte Carlo and quantum Monte Carlo diagonalization methods, and a greatly improved direct diagonalization approach, have greatly expanded the range of shell-model applications. It is now possible to take the full fp-shell basis into account for the mass region A = 40-60. These calculations require, as input, interactions (G matrices) for extremely large model spaces. They are obtained from modern nucleon-nucleon potentials, with appropriate (e.g. monopole term) corrections. Ultimately, they will be obtained in a model-independent way by fitting to observed energy levels. The direct diagonalization approach provides detailed wave functions from which one can obtain information important for astrophysics: the masses of nuclei far from stability, Gamow-Teller strength functions, spectroscopic factors, and level densities. For example, the fp-shell calculations have provided the weak interaction rates that are needed to understand stellar-core collapse and supernova formation. Monte Carlo methods have been used in the sdg-shell for nuclei in the ^{100}Sn region. One can expect much more progress in this direction in the future.

For light nuclei one can consider "no-core" model spaces that take into account up to about 10 harmonic-oscillator shells. The interactions for these calculations are obtained from nucleon-nucleon scattering G-matrix elements, and improved methods, using the Bloch-Horowitz method to describe effective many-body interactions, are now being explored. The shell-model codes OXBASH and ANTOINE are available for use by researchers, and other more powerful codes are being developed.

Challenges for nuclear theory from the astrophysics viewpoint

- To determine the cross sections for important reactions in few-body systems. Could the ^{3}He + p → ^{4}He+ ν_e + e^+ cross section possibly be large enough (over 10 times the present estimate) to explain the high-energy flux in Super-Kamiokande?

- To develop a continuum shell-model method to deal with the contributions of unbound orbitals as one nears the neutron drip line. Are the shell splittings reduced (quenched) for n-rich nuclei? Can one find signatures of this quenching in the observable regions, thereby giving guidance to experimental studies?

- To determine the weak interaction response of nuclei for both neutron and proton rich nuclei. Perform detailed large basis shell model calculations to provide the electron capture and beta-decay strengths necessary to simulate the evolution of the supernova core and the r-process. Obtain the global neutrino interaction rates for both charged and neutral-currents necessary to assess the role of neutrino induced reactions and energy transfer in supernovae – transitions to unstable states should be included.

- To obtain appropriate asymptotic forms of many-body wave functions for reaction calculations and their comparison with experimental results.

- To develop a reliable temperature, density, N/Z dependent equation of state appropriate for use in various astrophysical sites, including neutron stars and supernovae.

Application to nuclear astrophysics

The interaction of nuclear theory with astrophysical theory is complicated. In the context of nuclear physics, when a nuclear structure calculation has been completed it is compared with experiment, and if necessary revisited taking the experimental results into account. In the context of nuclear astrophysics several additional steps are necessary. Since information on a broad range of nuclei is often required, development of an extrapolation or interpolation procedure may be necessary if the theoretical calculations are difficult or time consuming. The theory must then be validated by comparison to an appropriate range of experimental data. If its predictions are not as accurate as is required, the theory must be “normalized” to experiment in an appropriate fashion. Then, relevant cross sections or lifetimes must be calculated. This may involve straightforward computations of matrix elements or strengths. Or it may, for example, involve computations of inelastic neutrino scattering cross sections, or of radiative capture cross sections for isolated resonances, or the use of statistical approaches. Finally, these results must be expressed in terms of temperature dependent astrophysical rates.

The complexity of the above task means that considerable thought must be given to deciding when a new theory is sufficiently developed and sufficiently different from its predecessors to warrant revising the reaction rates. It also means that a coordinated effort must be made to address these problems – the theoretical strength at any single institution will be insufficient. Perhaps such a coordinated effort could help to ensure that the structure of the rate calculations and of the astrophysics programs that use them is such that improved theoretical and experimental information can be incorporated with relative ease.

An important role of theory in nuclear astrophysics lies in providing guidance on the required measurements. Many astrophysical processes involve networks of reactions and decays; often it is far from clear whether a change in a particular nuclear property will have an impact in the astrophysical scenario. At other times, reactions occur under conditions of thermal equilibrium, and only the masses of nuclei and beta-decay strength are important. The importance of a given reaction may depend on the specific situation, for example on the mass of the star involved. Finally, it is clear that this process must be iterated, since changes in some rates may affect the importance of others. It will not be simple to provide guidance on the precision with which a given rate must be known, but it is essential to provide this guidance so that our experimental and theoretical efforts can proceed efficiently.

A related issue is the need to ensure that the available data are quickly and conveniently accessible to those involved in using them directly for the generation of rates for astrophysical models, or as part of the evaluation process for theoretical calculations. Else much of the effort involved in making these difficult measurements will be wasted.