Computation of neutrino transport in core-collapse supernovae

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Heating/cooling rates depend on accurate evolution of neutrino distributions.

\[ f(t, x, p) \]
Convection, rotation, and magnetic fields all come into play.
SASI: The stationary accretion shock is intrinsically unstable and could generate phenomena traditionally attributed to progenitor rotation.

Aspherical explosion morphology

Magnetic field generation

Pulsar spin

Endeve et al. (2008)

Blondin, Mezzacappa, and DeMarino (2003)

Blondin and Mezzacappa (2007)
Confidence in the simulations derives from successful confrontations with observational data.

- Launch of an explosion
- Neutron star mass, magnetic field, and kick velocity
- Composition of ejecta
- Explosion morphology
- Neutrino signals
- Gravitational wave signals
Two observables, beyond explosion, related to $\nu$ transport:

Accretion continues until the stalled shock is reinvigorated: relation between *neutron star mass* and *delay to explosion*

The abundance of nuclei with a closed shell of 50 neutrons

The electron fraction...

\[
Y_e \equiv \frac{n_{e^-} - n_{e^+}}{n_{\text{baryons}}} \approx \frac{n_{\text{proton}}}{n_{\text{proton + neutron}}}
\]

...is set by $\nu$ interactions:

\[
\nu_e + n \leftrightarrow p + e^- \\
\bar{\nu}_e + p \leftrightarrow n + e^+
\]
In its full glory, neutrino transport leads to exascale simulations.

Fluid dynamics in 3D came to greater maturity at the terascale.

A handful of variables depending on time $t$ and position $x$.

The full dimensionality of kinetic theory is 3D+3D.

The phase space distribution function $f(t, x, p)$ depends in addition on momentum $p$.

Assuming $\sim 100$ mesh cells in each momentum space dimension, a 3D+3D problem is about $10^6$ times larger than a 3D problem.

Terascale ($10^{12}$) $\rightarrow$ Exascale ($10^{18}$)
The distribution function obeys the Boltzmann equation.

\[ \partial_t f + v \cdot \nabla_x f + \dot{p} \cdot \nabla_p f = C[f] \]

- \( f(t, x, p) \) changes in \( t \)...
- ...as particles move away from \( x \) with velocity \( v \)...
- ...and their momenta \( p \) (speed and direction) are gently changed by long-range external forces \( \dot{p} = F \), typically electromagnetic or gravitational...
- ...and their \( p \) are abruptly changed by the short-range forces of particles’ point collisions.
The ‘stiffness’ of the Boltzmann equation introduces a computational challenge.

\[
\frac{f^{n+1} - f^n}{\Delta t} = \partial_t f + \mathbf{v} \cdot \nabla_x f + \mathbf{p} \cdot \nabla_p f = C[f]
\]

- **Dense couplings in** \( \mathbf{p} \)
  - ‘Implicit’ evaluation at \( f^{n+1} \)
  - \( \sim N_x N_p^{(2-3)} \)

- **Near-neighbor couplings in** \( \mathbf{x} \)
  - ‘Explicit’ evaluation at \( f^n \)
  - \( \sim N_x N_p \)

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The high dimensionality of phase space, addressed by neutrino transport, contributes to the exascale nature of the problem.

As an example, consider how easily the inversion of dense blocks arising from momentum space coupling can exhaust exascale resources.

\[ N_{\text{FLOP}} \sim N_t \ N_{\text{iterations}} \ N_x \ N_p^2 \]

\[ T_{\text{wall}} = \frac{N_{\text{FLOP}}}{\epsilon_{\text{FLOP}} \ R_{\text{FLOP}}} \]

\[ N_p = N_\nu \ N_E \ N_\theta \ N_\phi \]

\[ T_{\text{wall}} \sim 7 \ \text{weeks} \left( \frac{N_t}{10^6} \right) \left( \frac{N_{\text{iterations}}}{20} \right) \left( \frac{N_x}{10^6} \right) \left( \frac{N_p}{10^5} \right)^2 \left( \frac{R_{\text{FLOP}}}{10^{18} \ \text{s}^{-1}} \right)^{-1} \left( \frac{\epsilon_{\text{FLOP}}}{0.05} \right)^{-1} \]
Full neutrino transport would enable simulations of groundbreaking physical realism.

Current models on current machines cannot address phase space (position $x$ + momentum $p$) in its full dimensionality.

Reduction of dimensionality implies an unavoidable recourse to approximations and assumptions.

The full dimensionality of phase space could be addressed through adaptive mesh refinement (AMR) in momentum space.

AMR in space is a somewhat familiar technology, but AMR in momentum space will be new and powerful (recall $\sim N_x N_p^{(2-3)}$)

Support for the development of AMR in full phase space would facilitate exascale core-collapse supernova simulations.

Sophisticated algorithms developed on small scales will need to be ‘industrialized’ for deployment on leadership-class architectures.
How can we understand the explosion mechanism and test it through nucleosynthesis?

Neutrino transport, critical to the explosion mechanism, also impacts nucleosynthesis by setting $Y_e$.

**Large nuclear networks** (see talk by Raph Hix) and **neutrino transport** are the two most computationally intensive aspects of core-collapse supernova simulations.

Neutrino transport is an exascale problem because of the **high dimensionality** of $f(t, x, p)$ and the need for **implicit solution** of interactions, which scales as $\sim N_x N_p^{(2-3)}$.

AMR in momentum space and hardware accelerators can open access to the full dimensionality of momentum space by enabling inversion of larger dense blocks.

Transport with flavor mixing is particularly sensitive to $N_p$. 

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